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ANOMALOUS DIMENSION of TWIST FOUR GLUONIC OPERATOR.

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ABSTRACT

The anomalous dimension of the twist four gluonic operator (γ_4) in deep inelastic scattering is calculated in the double log approximation of perturbative QCD. It turns out that at $N \rightarrow 1$ the value of $\gamma_4(N-1)$ is close to $2\gamma_2(\frac{N-1}{2})$ (γ_2 is the anomalous dimension of the leading twist operator) but it is larger by factor $(1+\delta^2)$ where $\delta^2 \sim 10^{-2}$. It means that at $N \rightarrow 1$ (or $x_B \rightarrow 0$) the contribution of the higher twist operator becomes very important and gives rise to a screening correction to the deep inelastic structure function.

1. The main goal of the paper.

Let me start by recalling the main steps of our theoretical approach to deeply inelastic scattering:

1. We introduce the moments of the deep inelastic structure function,namely

$$M(\omega,r) \;=\; \int_0^1 x_B^{N-1} dx_B\, x_B G(x_B,Q^2) \;=\; \int_0^\infty e^{\omega y} dy [x_B G(x_B,Q^2)] \;, \qquad (1)$$

where $\omega = N-1$, $y = \ln(1/x_B)$ and $r = \ln(Q^2/Q_0^2)$.

2. Each moment is given as Wilson Operator Product Expansion in the form:

$$M(\omega,r) \;=\; C_2(\omega,r)\langle p|O^{(2)}|p\rangle \;+\; \frac{1}{Q^2}C_4(\omega,r)\langle p|O^{(4)}|p\rangle \;+\; \ldots\frac{1}{Q^{i-2}}C_i(\omega,r)\langle p|O^{(i)}|p\rangle \;\ldots \qquad (2)$$

where C_i is the coefficient function and $\langle p|O^{(i)}|p\rangle$ is the matrix element of the twist i operator (see ref.[1] for details).

3. It is well known from the renormalization group approach that a coefficient function C_i behaves as

$$C_i \;\propto\; e^{\gamma_i(\omega) r} \qquad (3)$$

where γ_i is the anomalous dimension of the twist i operator [1].

4. Now we neglected all high twist contributions (all terms in eq. (1) except the first one) assuming that they are small at large value of Q^2 due to the factor $\frac{1}{Q^{i-2}}$ in front.

[1] For simplicity we consider here the case of fixed α_s.

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5. The anomalous dimension of the leading twist contribution can be calculated using GLAP evolution equation [2] and it is equal to

$$\gamma_2(\omega) \; = \; \frac{N_c \alpha_s}{\pi \omega} \quad at \quad \omega \rightarrow 0 \tag{4}$$

6. The specific contribution to the value of the anomalous dimension of high twist operator that originates from the exchange of many 'leading twist ladders' in t-channel was found in the GLR paper [3] . It gives

$$\gamma_{2n}(\omega) \; = \; n \gamma_1 (\frac{\omega}{n}) \, . \tag{5}$$

It is very instructive to consider the twist four contribution to illustrate the above statement. The two ladder exchange leads to the following contribution

$$C_4(\omega, r) \langle p | O^{(4)} | p \rangle \; = \; \int \frac{d\omega'}{2\pi i} C_2(\omega', r) C_2(\omega - \omega', r) \langle p | O^{(4)} | p \rangle \; \propto \; \int d\omega' e^{\gamma_2(\omega - \omega') r \, + \, \gamma_2(\omega') r} \, . \tag{6}$$

The integral has an obvious saddle point $\omega' \; = \; \omega/2$, so $C_4 \; \propto \; e^{2\gamma_2(\frac{\omega}{2}) r}$. Thus $\gamma_4 \; = \; 2\gamma_2(\frac{\omega}{2})$.

2. The result.

Recently Bartels [4] and Levin,Ryskin and Shuvaev [5] have made the next step in understanding the high twist contribution to eq. (1) and both groups calculated the anomalous dimension of the twist four gluon operator, using a completely different techniques. It turns out that the value of the anomalous dimension is equal to

$$\gamma_4 \; = \; 2 \gamma_2 (\frac{\omega}{2}) [1 \; + \; \delta^2] \; = \; \frac{4 N_c \alpha_s}{\pi \omega} \cdot [1 \; + \; \delta^2] \tag{7}$$

where $\delta^2 \sim (\frac{1}{N_c^2 - 1})^2 \approx 10^{-2}$ is very small.

The most important outcome of this calculation is the fact that we **cannot trust the GLAP evolution equation** in the region of small ω (or large $\ln(1/x_B)$). Indeed for $\omega < \omega_{cr}$ the twist four contribution in eq. (1) becomes larger than the leading twist one. The value of ω_{cr} can be found from the equation

$$\gamma_2(\omega_{cr}) \; = \; \frac{N_c \alpha_s}{\pi \omega_{cr}} \; = \; -1 \; + \; \gamma_4(\omega_{cr}) \; = \; -1 \; + \; \frac{4 N_c \alpha_s}{\pi \omega_{cr}} [1 + \delta^2] \, .$$

Of course we could arrive at the same conclusion using GLR approach but now we proved this statement considering the whole set of Feynman diagrams instead of the two ladder contribution that the GLR paper took into account [3].

Difference. In two papers the quite different approaches have been used. Bartels' one is based on reggeon technique in multireggeon kinematical region and s- channel and t-channel unitarity. We sum up the Feyman diagrams directly in double log approximation of QCD with integration over longitudinal coordinates accordingly the rules formulated in ref. [6]. We got the different results, namely $\delta_{Bartels} \; = \; 2 \; \delta_{our}$.It is not clear at all who is right, but I want to stress that this difference is intimtaly connected with the space - time structure of the parton cascade in QCD. So when we will understand this difference we certainly will have to say more about cascade evolution in QCD.

Good news. I would like to discuss first several lessons that we have learned from this calculation:

1. Eq. (7) confirms the main hypothesis of ref. [3] which is that the small x_B behaviour of the deep inelastic structure function is determined by the exchange of many Pomerons in t-channel and their interaction.

2. The smallness of δ mentioned above reflects the smallness of pomeron - pomeron interaction which is nonplanar and proportional to $\frac{1}{N_c^2-1}$.

3. Strictly speaking the pomeron-pomeron interaction was not taken into account in the GLR - equation.However the good news is the fact that the correction to the GLR equation is so small that it gives a noticeable contribution only at ultra high energies.

Bad news. However the main theoretical conclusion from this exercise looks rather pessimistic because it was shown that QCD cannot cure the old problem of the reggeon approach that was pointed out in ref. [6], namely, the fact that pomeron cannot be the correct first approximation to high energy interaction of virtual photon with a hadron at least in perturbative QCD. In other words the pomeron - pomeron interactions turns out to be attractive and the system of many pomerons cannot be stable. Of course we made only the first step to study the above problem in a self consistent way and the next one will be to consider the value of anomalous dimension of even higher twist operators, but there is a priori no reason to think that the specific coherence effects in QCD will be able to help us for the twist $n > 4$ operators.

3. The evolution equation.

I think it is very instructive to understand the physical meaning of this result in terms of the evolution equation for the parton cascade . We can suggest the generalization of the GLR evolution equation taking into account the interaction of the pomerons for the twist four operator. It looks like a system of two equations. which are simpler to write down in the integro - differential form in the DLA of perturbative QCD.

$$\frac{\partial^2 xG(x,Q^2)}{\partial \log\frac{1}{x}\,\partial\log Q^2} \;=\; \alpha_s xG(x,Q^2) \;-\; \alpha_s^2\cdot\frac{4\pi N_c^2}{(N_c^2-1)Q^2}\cdot(x^2G^{(2)}(x,Q^2)). \tag{8}$$

and

$$x^2G^{(2)}(x,Q^2) \;=\; \frac{9}{8\pi R^2}\cdot(xG(x,Q^2))^2 +$$
$$+\frac{2N_c\delta\alpha_s}{\pi}\int_x^1\frac{dx'}{x'}\int^{Q^2}\frac{dk^2}{k^2}\int_{x'}^1\frac{dx"}{x"}\frac{x"}{x}G(\frac{x}{x"},\frac{Q^2}{k^2})\frac{x'}{x}G(\frac{x}{x'},\frac{Q^2}{k^2})x'^2G^{(2)}(x',k^2)\;-$$
$$-\frac{\alpha_s^2 g_{2\to3}}{R^2}\int_x^1\frac{dx'}{x'}\int^{Q^2}\frac{dk^2}{k^4}(\frac{x'}{x})^2G^{(2)}(\frac{x}{x'},\frac{Q^2}{k^2})x'^2G^{(2)}(x',k^2)x'G(x',k^2). \tag{9}$$

The first equation is the equation for the parton cascade which is written with better accuracy than the GLR equation [3], since the probability for two partons to have the same x and Q^2 ($G^{(2)}$) was introduced (see ref. [7] where it was done first for details). The second one is new. From this equation you can see that in the GLR equation we assumed that there was no correlation between two gluons except the fact that they are distributed in the hadron disc of the radius R. However the second equation shows that it is not true and the correlation increases until the screening correction enters to the game and this growth will be stopped due to them. We can express the same physics saying that the correlation radius between two gluons increases with x_B and they create a more compact system then the hadron. In eq. (9) we assumed that the probability of three parton interaction $G^{(3)} \propto G^{(2)} \cdot G$.

4.The effective two dimension theory.

The next step in our understanding as has been discussed should be the calculation of γ_{2n} at large value of n. It is really amazing that we can reduce the complicated job of summing all Feynman diagrams in DLA of QCD to a very elegant two dimension theory. Indeed if we introduce the new variables $t = y - r$ and $R = y + r$ and field $\varphi(x)$ where $x_\mu = (t, R)$ we are able to rewrite the function $x^n G^{(n)}$ in the form $x^n G^{(n)} = T(\varphi^{+n}(x)\varphi^{-n}(0))$. The Lagrangian of the effective theory looks as follows:

$$L = \frac{\partial \varphi^+(x)}{\partial x_\mu} \frac{\partial \varphi^-(x)}{\partial x_\mu} + m^2 \varphi^+(x)\varphi^-(x) + \lambda \varphi^+(x)\varphi^+(x)\varphi^-(x)\varphi^-(x) \, , \tag{10}$$

where $m^2 = \frac{N_c \alpha_s}{\pi}$ and $\lambda = 2m^2\delta > 0$. All our difficulties is seen from the Lagrangian, namely we have the system of effective particles with attractive forces. Our experience tells us that such forces lead to a fall toward the centre and an unstable system. However we have a two dimensional theory which could have a phase transition. In this case we could have some hope to find reasonable solution of QCD in the region of small x_B.

5. Conclusions.

1. The GLAP evolution equation cannot describe the deep inelastic structure function in the region of small x_B since the high twist operators give larger contribution than the leading one.

2. The nonlinear GLR evolution equation is proven to be correct in QCD with large number of colours ($N_c \rightarrow \infty$), but it is not clear at all how well it will be able to describe the real situation. The small correction to the value of the anomalous dimension of the twist four operator encourages us, however the urgent problem is to calculate the anomalous dimension for twist n operator with $n \gg N_c$.

3. We need systematic study of high twist contribution and not only in the region of small x_B.

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7. References

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